

Exercise 54

For the following exercises, evaluate the expressions, writing the result as a simplified complex number.

$$\frac{(1 + 3i)(2 - 4i)}{(1 + 2i)}$$

Solution

Simplify the given expression.

$$\begin{aligned}\frac{(1 + 3i)(2 - 4i)}{(1 + 2i)} &= \frac{2(1 + 3i)(1 - 2i)}{1 + 2i} \\ &= \frac{2(1 - 2i + 3i - 6i^2)}{1 + 2i} \\ &= \frac{2(1 + i + 6)}{1 + 2i} \\ &= \frac{2(7 + i)}{1 + 2i} \\ &= \frac{2(7 + i)}{1 + 2i} \times \frac{1 - 2i}{1 - 2i} \\ &= \frac{2(7 + i)(1 - 2i)}{(1 + 2i)(1 - 2i)} \\ &= \frac{2(7 - 14i + i - 2i^2)}{1 - 2i + 2i - 4i^2} \\ &= \frac{2(7 - 13i + 2)}{1 - 4(-1)} \\ &= \frac{2(9 - 13i)}{5} \\ &= \frac{18 - 26i}{5} \\ &= \frac{1}{5}(18 - 26i) \\ &= \frac{18}{5} - \frac{26}{5}i\end{aligned}$$